Engineering Notes

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Pole Placement with Output Feedback

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Introduction

IN practice, the control system designer rarely has the freedom to feed back the entire state of the system to achieve the desired system performance. Thus, an important problem from a practical standpoint is the determination of constant, output feedback gains for the control of systems with inaccessible states. The method discussed here is of the pole placement type, but is depends only on the minimization of a function of a distance between the desired poles and the poles of the closed-loop system.

Output feedback, where each output is fed to each input, could still be quite complex, but the controller can be simplified by imposing structural constraint on it and thereby eliminating some of the gains operating on accessible outputs. It is well known that feedback gains do not contribute uniformly to improve total system performance. Thus, many gains can be termed nonproductive because of the minimal effect they have on system performance. Consequently, judicious selection of active gains can result in a significant reduction of controller complexity, and it can be achieved with minimal performance sacrifice.

It is the purpose of this Note to outline a set of conceptual and computational pole placement procedures that resolve the problem of providing output feedback applicable to arbitrary open-loop systems. In addition, the designer can impose structural constraints on the controller that eliminate some preselected gains corresponding to accessible output.

Problem Formulation

Consider a linear, time invariant, dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$, and A, B, and C are real, constant matrices of compatible order. Upon application of the output feedback control $u(t) = -Fy(t)$, where F is an $m \times r$ constant matrix, the resulting closed-loop system is described by

$$\dot{x}(t) = (A - BFC)x(t) \tag{3}$$

Let the set of eigenvalues of the matrix $A - BFC$, which determine the dynamics of this closed-loop system, comprise a proper spectrum $\sigma(A - BFC)$. Thus, given a set of self-conjugate scalars $M = \{\mu_i\}$, it is desired to find F such that

$$\sigma(A - BFC) = M \tag{4}$$

Additionally, only certain of the mr elements of the matrix F will be allowed to participate in the control process. The nonparticipating elements will be fixed at zero in order to illustrate the simplification achieved in the controller structure.

Problem Solution

Let $\{\lambda(f)\}$ be the set of eigenvalues of the matrix $A - BFC$, for an arbitrary matrix F, and $\{\mu_i\}$ be the set of eigenvalues for the desired system, where $\lambda_i, \mu_i \in C^l$.

Consider the $\{\lambda_i\}$ in conjunction with the $\{\mu_i\}$. These sets represent the locations of the closed-loop poles for the actual system and the target system, respectively. We shall proceed to define a measure of the separation between the two sets.

In order to do so an ordering is imposed on each of the sets. That is, we sort the eigenvalues by sweeping the real axis from $-\infty$ and increasing the subscripts as we proceed to $+\infty$. Whenever a complex conjugate pair, or a triple alignment is encountered, the eigenvalues are ordered according to the positivity of their imaginary parts. The one with the most positive imaginary part is assigned the smaller subscript of the aligned group.

Having imposed this ordering on the actual and desired sets of eigenvalues, the separation between the sets will be measured, as shown in Fig. 1, namely by a sum of squares of the lengths l_i $(i = 1,...,n)$. The objective is to minimize, with respect to F, a function which is a metric between the actual closed-loop poles and the target poles. This function is defined as

$$J(F) = \sum_{i=l}^{n} l_i^2 \tag{5}$$

where

$$l_i^2 = [\mathrm{Re}(\mu_i) - \mathrm{Re}(\lambda_i)]^2 + [\mathrm{Im}(\mu_i) - \mathrm{Im}(\lambda_i)]^2 \tag{6}$$

Algorithm Description

Based on the preceding discussion, an algorithm for placing the poles of a linear, time invariant system with output feedback and additional controller structural constraints will

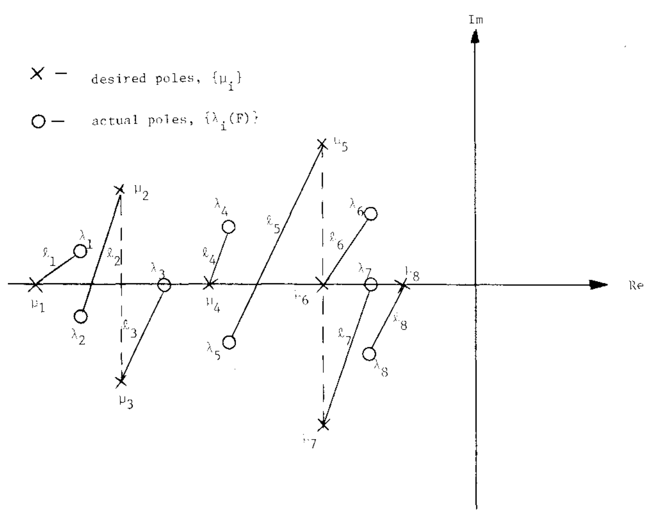

Fig. 1 Eigenvalue separation measurement.

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be briefly described:

1) Input A,B,C,F_0, $\{\mu_i\}$, allowable f_{ij} ($F_0=\{0\}$ is permissible).
2) Compute $\lambda(A-BF_0C)$.
3) Perform ordering on $\{\lambda_i\}$ and $\{\mu_i\}$.
4) Compute

$$J(F)=\sum_{i=1}^{n}\{[\mathrm{Re}(\mu_i)-\mathrm{Re}(\lambda_i)]^2+[\mathrm{Im}(\mu_i)-\mathrm{Im}(\lambda_i)]^2\}.$$

5) Find F^* that minimizes $J(F)$ by use of subroutine ZXMIN of IMSL.[1]
6) Compute $\lambda(A-BF^*C)$ and print F^* and the final λ_i.

Numerical Example

The method presented will be illustrated through its application to a simplified flight control problem. The inner-loop lateral axis design problem, taken from Ref. 2 will be used to demonstrate the method. The aircraft dynamics for this problem are given by Eqs. (1) and (2), where

$$x=\begin{bmatrix} p_s \\ r_s \\ \beta \\ \delta_a \\ \delta_r \\ \phi \end{bmatrix} \begin{matrix} \text{stability axis roll rate} \\ \text{stability axis yaw rate} \\ \text{angle of sideslip} \\ \text{aileron deflection} \\ \text{rudder deflection} \\ \text{bank angle} \end{matrix} \tag{7}$$

$$u=\begin{bmatrix} \delta_{rc} \\ \delta_{ac} \end{bmatrix} \begin{matrix} \text{rudder command} \\ \text{aileron command} \end{matrix} \tag{8}$$

where A and B are given by

$$A=\begin{bmatrix} -0.746 & 0.387 & -12.9 & 6.05 & 0.952 & 0 \\ 0.024 & -0.174 & 0.4 & -0.416 & -1.76 & 0 \\ 0.006 & -0.999 & -0.058 & -0.0012 & 0.0092 & 0.0369 \\ 0 & 0 & 0 & -5.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10.0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$$B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 10.0 \\ 20.0 & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

$$C=[I_5 \;\; 0] \tag{11}$$

The elements of the initial feedback matrix F are set at zero. The desired poles are specified as

$\lambda_1=-200$ actuator

$\lambda_2=-100$ actuator

$\lambda_3=-4$ roll subsidence

$\lambda_4=-1.77+j\,1.77$ Dutch roll

$\lambda_5=-1.77-j\,1.77$ Dutch roll

$\lambda_6=-0.005$ spiral mode (12)

Two allowable configurations of the feedback matrix will be used. The first is

$$F_1=\begin{bmatrix} 0 & f_{12} & f_{13} & 0 & f_{15} \\ f_{21} & 0 & 0 & f_{24} & 0 \end{bmatrix} \tag{13}$$

The matrix that minimized the function $J(F)$, Eq. (7), was found to be

$$F_1^*=\begin{bmatrix} 0 & -17.81 & 36.65 & 0 & 9.66 \\ 5.7 & 0 & 0 & 9.85 & 0 \end{bmatrix} \tag{14}$$

The poles achieved by use of Eq. (14) in the closed-loop matrix $A-BFC$ are

$\lambda_1=-200$

$\lambda_2=-100$

$\lambda_3=-4$

$\lambda_4=-1.77+j\,1.77$

$\lambda_5=-1.77-j\,1.77$

$\lambda_6=-0.061$ (15)

All of the desired poles were matched perfectly except for the spiral mode (λ_6). This is not surprising, as bank-angle feedback ϕ is precluded by the structure of C of Eq. (11).

The second configuration, allowing for feedback of angle of sideslip β to the aileron, is given by Eq. (1), but, f_{23} is also activated. The F matrix of this structure that minimized the function $J(F)$ was found to be

$$F_2^*=\begin{bmatrix} 0 & -18.14 & 37.19 & 0 & 9.66 \\ 5.587 & 0 & -4.17 & 9.84 & 0 \end{bmatrix} \tag{16}$$

The resulting poles of the closed-loop system were precisely those of Eq. (12). So, perfect pole placement was possible with the addition of element f_{23} to the feedback matrix structure.

Conclusions

An approach for generating constant output feedback gains has been presented. In addition, this approach allows the designer to simplify the controller structure by eliminating gains, corresponding to accessible outputs, but which are known or found to be nonproductive in achieving the desired system performance. Algorithmic structure as well as a numerical example have been presented.

References

[1] "Computer Subroutine Libraries in Mathematics and Statistics," International Mathematical and Statistical Libraries, Inc., Houston, Texas, IMSL LIB-006, April 1977.

[2] Harvey, C.A. and Stein, G., "Quadratic Weights for Asymptotic Regulator Problems," *IEEE Transactions on Automatic Control*, Vol. AC-23, June 1978, pp. 378-387.